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Pion distribution amplitude from holographic QCD and the electromagnetic form factor $F_\pi(Q^2)$

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The holographic QCD prediction for the pion distribution amplitude (DA) $\varphi_{hol}(u)$ is used to compute the pion spacelike electromagnetic form factor $F_\pi(Q^2)$ within the QCD light-cone sum rule method. In calculations the pion's renormalon-based model twist-4 DA, as well as the asymptotic twist-4 DA are employed. Obtained theoretical predictions are compared with experimental data and with results of the holographic QCD.

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I. INTRODUCTION

Recently holographic dual models of QCD were proposed [1, 2, 3, 4, 5, 6, 7] and applied for investigations to the hadronic physics. This approach is based on the AdS/CFT correspondence [8], and is aiming to construct dual models of QCD in 5-dimensional Anti-de Sitter (AdS) space. Invented models incorporate important features of QCD, as confinement and chiral symmetry breaking, and allow one to calculate numerous hadronic properties including masses, decay constants, couplings of various mesons.

The holographic models of QCD were employed for investigation of mesons electromagnetic form factors (FFs), as well. Calculations of the pion and ρ-meson FFs in Refs. [9, 10, 11, 12, 13] were carried out in the framework of two models, namely in hard-wall and soft-wall ones depending on the procedure to impose the infrared cutoff on the fifth (holographic) dimension of the AdS space.

It is remarkable, that the holographic QCD (HQCD) predicts also mesons' valence wave functions thus providing analytic approximation to QCD [12]. By this way, the HQCD prediction for the pion distribution amplitude (DA) $\varphi_{hol}(u)$ was derived. It differs from the pion asymptotic DA $\varphi_{asy}(u)$ found from the perturbative QCD (PQCD) evolution [14], and due to the broader shape should increase the magnitude of the leading twist QCD prediction for the pion electromagnetic FF.

In the present work the DA $\varphi_{hol}(u)$ is used to compute the pion spacelike electromagnetic form factor in the context of the QCD light-cone sum rule (LCSR) method with twist-6 accuracy by taking into account $O(\alpha_s)$ order correction to the leading twist term. The twist-4 DA of the pion, necessary to determine the twist-4 contribution to the FF, is derived in the context of the renormalon method. The twist-4 contribution is also modeled employing the asymptotic form of the pion twist-4 DA following from the conformal expansion.

The paper is structured as follows: In Sec. II general expressions for the form factor $F_\pi(Q^2)$ in LCSR method are presented. Section III is devoted to calculation of the pion twist-4 DA employing the renormalon approach. In Sec. IV we present results of numerical computations and compare them with the experimental data and predictions of the holographic QCD. Section V contains our conclusions.

II. THE PION ELECTROMAGNETIC FORM FACTOR IN THE QCD LCSR METHOD

It is known that the QCD LCSR method is one of the powerful tools to evaluate nonperturbative components of exclusive quantities [15]. The LCSR expression for the pion electromagnetic FF was derived in Refs. [16, 17, 18]: with the twist-6 accuracy it has the following form

$$F_\pi(Q^2) = F_\pi^{(2)}(Q^2) + F_\pi^{(2,\alpha_s)}(Q^2) + F_\pi^{(4)}(Q^2) + F_\pi^{(6)}(Q^2), \tag{2.1}$$

where $Q^2 = -q^2$, q being the four-momentum of the virtual photon in the process $\gamma^*\pi^\pm \to \pi^\pm$. In Eq. (2.1) $F_\pi^{(n)}(Q^2)$ is the twist-n contribution to the FF; $F_\pi^{(2,\alpha_s)}(Q^2)$ is $O(\alpha_s)$ order correction to the twist-2 part.

The leading twist (twist-2) light-cone sum rule for $F_\pi(Q^2)$ at the leading order is given by the expression

$$F_\pi^{(2)}(Q^2) = \int_{u_0}^1 du\varphi^{(2)}(u,\mu_F^2)\exp\left[-\frac{\overline{u}Q^2}{uM^2}\right], \tag{2.2}$$

where

$$u_0 = \frac{Q^2}{s_0+Q^2}. \tag{2.3}$$

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In Eqs. (2.2) and (2.3) $\varphi^{(2)}(u,\mu_F^2)$ is the pion leading twist DA; s_0 is the duality interval, M^2 is the Borel variable and $\overline{u} \equiv 1-u$.

The $O(\alpha_{\rm s})$ order correction to the leading twist term was obtained in Ref. [17]. Then the whole twist-2 contribution to the form factor

$$\mathbf{F}_\pi^{(2)}(Q^2) = F_\pi^{(2)}(Q^2) + F_\pi^{(2,\alpha_{\rm s})}(Q^2)$$

can be written as the sum of the soft and hard parts

$$\mathbf{F}_\pi^{(2)}(Q^2) = \int_0^1 du\varphi^{(2)}(u,\mu_F^2)\left[\Theta(u-u_0)F_{soft}^{(2)}(u,M^2,s_0)\right.$$

$$\left. +\Theta(u_0-u)F_{hard}^{(2)}(u,M^2,s_0)\right]. \qquad (2.4)$$

The soft part of the LCSR contains entirely the leading order twist-2 contribution and some piece of the next-to-leading order correction to the twist-2 term. Stated differently, the expression presented in Eq. (2.2) is the pure soft contribution to the FF. The twist-2 term (2.4) is linear in the pion DA. Its hard component at high momentum transfers $Q^2 \gg s_0$ leads to the PQCD prediction for the FF quadratic in DA [17]. For example, choosing as $\varphi^{(2)}(u,\mu_F^2)$ the pion asymptotic DA $\varphi_{asy}(u)$, from the hard part of the LCSR one regains the well-known asymptotic prediction of the PQCD [14, 19]

$$F_\pi^{asy}(Q^2) = \frac{8\pi\alpha_{\rm s}(Q^2)f_\pi^2}{Q^2},$$

with $f_\pi = 0.132$ GeV being the pion decay constant.

Further details of calculations and explicit expressions for $F_{soft}^{(2)}(u,M^2,s_0)$ and $F_{hard}^{(2)}(u,M^2,s_0)$ can be found in Ref. [17].

The twist-4 term $F_\pi^{(4)}(Q^2)$ is determined as [18]

$$F_\pi^{(4)}(Q^2) = \int_{u_0}^1 du \frac{\phi_4(u,\mu_F^2)}{uM^2}\exp\left[-\frac{\overline{u}Q^2}{uM^2}\right]$$

$$+\frac{u_0\phi_4(u_0,\mu_F^2)}{Q^2}e^{-s_0/M^2}. \qquad (2.5)$$

Here

$$\phi_4(u,\mu_F^2) = 2u\left[\frac{d}{du}\varphi_2^{(4)}(u,\mu_F^2) - \overline{u}\frac{d^2}{du^2}\varphi_2^{(4)}(u,\mu_F^2)\right], \qquad (2.6)$$

and $\varphi_2^{(4)}(u,\mu_F^2)$ is the pion two-particle twist-4 DA.

The factorizable twist-6 contribution to the LCSR was computed in Ref. [17] in terms of the quark condensate density

$$F_\pi^{(6)}(Q^2) = \frac{4\pi\alpha_{\rm s}(\mu_R^2)C_F}{3f_\pi^2Q^4}\left\langle 0\left|\overline{q}q\right|0\right\rangle^2, \qquad (2.7)$$

where $C_F = 4/3$ is the color factor.

In the QCD LCSR method for the factorization and renormalization scales the following values should be accepted:

$$\mu_F^2 = \mu_R^2 = \overline{u}Q^2 + uM^2. \qquad (2.8)$$

Equations (2.1)-(2.7) supplemented by the prescription (2.8) form a basis for investigation of the pion electromagnetic FF in the QCD LCSR method.

III. THE PION DISTRIBUTION AMPLITUDES

The light-cone two-particle distribution amplitudes of the pion are defined through the light-cone expansion of the matrix element

$$\left\langle 0\left|\overline{d}(x_2)\gamma_\nu\gamma_5\left[x_2,x_1\right]u(x_1)\right|\pi^+(p)\right\rangle = if_\pi p_\nu\int_0^1 due^{-iupx_1-i\overline{u}px_2}\left[\varphi^{(2)}(u,\ \mu_F^2) + \Delta^2\varphi_1^{(4)}(u,\ \mu_F^2) + O(\Delta^4)\right]$$

$$+\,if_\pi\left(\Delta_\nu(p\Delta) - p_\nu\Delta^2\right)\int_0^1 due^{-iupx_1-i\overline{u}px_2}\left[\varphi_2^{(4)}(u,\ \mu_F^2) + O(\Delta^4)\right], \qquad (3.1)$$

where $\varphi_1^{(4)}(u,\ \mu_F^2)$, $\varphi_2^{(4)}(u,\ \mu_F^2)$ are two-particle twist-4 DAs, $\Delta = x_1 - x_2$, and we use the notation $[x_2,x_1]$ for the Wilson line connecting the points x_1 and x_2.

The standard method to handle meson DAs is modeling them employing the conformal expansion. Then for the leading twist pion DA we get [14]

$$\varphi^{(2)}(u,\ \mu_F^2) = \varphi_{asy}(u)\left[1+\sum_{n=2,4..}^{\infty} b_n(\mu_F^2)C_n^{3/2}(u-\overline{u})\right]. \qquad (3.2)$$

Here $\varphi_{asy}(u)$ is the PQCD asymptotic DA of the pion

$$\varphi_{asy}(u) = 6u\overline{u},$$

and $C_n^{3/2}(\xi)$ are the Gegenbauer polynomials. The functions $b_n(\mu_F^2)$ determine the evolution of $\varphi^{(2)}(u,\, \mu_F^2)$ on the factorization scale μ_F^2 [14],

$$b_n(\mu_F^2) = b_n^0 \left[\frac{\alpha_s(\mu_F^2)}{\alpha_s(\mu_0^2)}\right]^{\gamma_n/\beta_0},$$

$$\gamma_n = C_F \left[1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1}\frac{1}{j}\right]. \tag{3.3}$$

In the above γ_n are the anomalous dimensions, μ_0^2 is the normalization scale, and $b_n^0 \equiv b_n(\mu_0^2)$. The expansion over the conformal spin can also be performed for the higher twist DAs (see, for example, Refs. [20, 21]).

An alternative way to find the higher twist DAs is the renormalon approach [22, 23]. The renormalon approach employs the assumption that the infrared renormalons in the leading twist coefficient functions should cancel the ultraviolet renormalons in the matrix elements of twist-4 operators in a relevant operator product expansion. Such cancellation was proved by explicit calculations in the case of the simple exclusive amplitude involving pseudoscalar and vector mesons [23]. It turned out that this is enough to find the full set of two- and three-particle twist-4 DAs of pseudoscalar and vector mesons in terms of their leading twist DAs. Higher twist DAs of some of the mesons were computed using the renormalon technique in the papers [24, 25, 26].

In the renormalon-based model the pion twist-4 DAs are given by the formulas [23]

$$\varphi_1^{(4)}(u,\mu_F^2) = \frac{\delta^2}{6}\int_0^1 dv\varphi^{(2)}(v,\mu_F^2)\left\{\frac{1}{v^2}\left[u + (v-u)\ln\left(1-\frac{u}{v}\right)\right]\theta(v>u) + \frac{1}{\overline{v}^2}\left[\overline{u} + (u-v)\ln\left(1-\frac{\overline{u}}{\overline{v}}\right)\right]\theta(v<u)\right\}, \tag{3.4}$$

$$\varphi_2^{(4)}(u,\mu_F^2) = -\frac{\delta^2}{6}\int_0^1 dv\varphi^{(2)}(v,\mu_F^2)\left[\left(\frac{u}{v}\right)^2\theta(v>u) + \left(\frac{\overline{u}}{\overline{v}}\right)^2\theta(v<u)\right]. \tag{3.5}$$

As is seen the renormalon model for the twist-4 DAs depends only on one free parameter δ^2. It is related to the matrix element of the local operator

$$\left\langle 0\left|\overline{d}\gamma_\nu ig\widetilde{G}_{\mu\rho}u\right|\pi^+(p)\right\rangle = \frac{1}{3}f_\pi\delta^2\left[p_\rho g_{\mu\nu} - p_\mu g_{\rho\nu}\right],$$

$$\delta^2(\mu_F^2) = \delta^2(\mu_0^2)\left[\alpha_s(\mu_F^2)/\alpha_s(\mu_0^2)\right]^{8C_F/3\beta_0}, \tag{3.6}$$

and at $\mu_0^2 = 1$ GeV^2 was estimated from various 2-point QCD sum rules [25, 27]

$$\delta^2(\mu_0^2) \equiv \delta_0^2 = 0.18 \pm 0.06 \text{ GeV}^2. \tag{3.7}$$

In Eq. (3.7) we use the latest available estimation for δ_0^2 [25]. In what follows we also do not show explicitly a dependence of the parameter δ^2 on the factorization scale μ_F^2.

In the case of the pion leading twist DA with two non-asymptotic terms ($b_2(\mu_F^2)$, $b_4(\mu_F^2) \neq 0$, and $b_n = 0, n > 4$) these higher twist distributions were calculated in Ref. [24] (see Erratum in Ref. [28]). Let us rewrite the twist-4 DA $\varphi_2^{(4)}(u,\mu_F^2)$ obtained in Ref. [24], and solely relevant for our present studies, in the compact form

$$\varphi_2^{(4)}(u,\mu_F^2) = \delta^2\left[u\overline{u} + u^2\ln u + \overline{u}^2\ln\overline{u} + 6b_2(\mu_F^2)\left(u^2\ln u + \overline{u}^2\ln\overline{u} + u\overline{u} + \frac{5}{3}u^2\overline{u}^2\right)\right.$$
$$\left. + 3b_4(\mu_F^2)\left(5u^2\ln u + 5\overline{u}^2\ln\overline{u} + 5u\overline{u} + \frac{77}{6}u^2\overline{u}^2 - 21u^3\overline{u}^3\right)\right]. \tag{3.8}$$

Then Eq. (2.6) is given by the expression:

$$\phi_4(u,\mu_F^2) = 2\delta^2\left\{-4u\overline{u} - 4u\overline{u}\ln\overline{u} + 2u(2u-1)\ln u + 4b_2(\mu_F^2)\left[-11u\overline{u} + 35u^2\overline{u} - 40u^3\overline{u} - 6u\overline{u}\ln\overline{u} + 3u(2u-1)\ln u\right]\right.$$

$$+b_4(\mu_F^2)\left[-137u\overline{u}+917u^2\overline{u}-3073u^3\overline{u}+4347u^4\overline{u}-2268u^5\overline{u}-60u\overline{u}\ln\overline{u}+30u(2u-1)\ln u\right]\Big\}\,. \tag{3.9}$$

The distribution amplitude of the pion obtained within the HQCD

$$\varphi_{hol}(u)=\frac{8}{\pi}\sqrt{u\overline{u}}, \tag{3.10}$$

in the renormalon approach leads to the following twist-4 DA

$$\varphi_2^{(4)}(u)=-\frac{8\delta^2}{3\pi}\left[u^2\left(\sqrt{\frac{\overline{u}}{u}}-\arctan\sqrt{\frac{\overline{u}}{u}}\right)\right.$$

$$\left.+\overline{u}^2\left(\sqrt{\frac{u}{\overline{u}}}-\arctan\sqrt{\frac{u}{\overline{u}}}\right)\right]. \tag{3.11}$$

As a result, for the twist-4 function $\phi_4^{hol}(u)$, we get

$$\phi_4^{hol}(u)=\frac{4\delta^2}{3\pi}\left[u(1+8u)\sqrt{\frac{\overline{u}}{u}}+(3-11u+8u^2)\sqrt{\frac{u}{\overline{u}}}\right.$$

$$\left.+8u(2u-1)\arctan\sqrt{\frac{\overline{u}}{u}}-16u\overline{u}\arctan\sqrt{\frac{u}{\overline{u}}}\right]. \tag{3.12}$$

The leading twist distributions $\varphi_{asy}(u)$ and $\varphi_{hol}(u)$ and the corresponding twist-4 DAs $\varphi_2^{(4)}(u)$ are depicted in Fig. 1. We see that as compared with $\varphi_{asy}(u)$ the holographic DA $\varphi_{hol}(u)$ is enhanced in the end-point domains. The difference between the twist-4 DAs shown in Fig. 1(b) is mild. As a result, the twist-4 functions $\phi_4(u)$ and $\phi_4^{hol}(u)$ that determine the twist-4 contribution to the form factor are close to each other (Fig. 2(a)). Nevertheless, we should note that the function $\phi_4^{hol}(u)$ is enhanced in the end-point region $u\to 1$. In Fig. 2(b) we demonstrate the variation of the form of $\phi_4^{hol}(u)$ depending on the chosen value of the parameter δ_0^2.

The twist-4 DA of the pion can be modeled in the context of the conformal expansion as well. The lowest conformal spin, i.e. the asymptotic form of the twist-4 DA $\varphi_2^{(4)}(u)$ was employed in Ref. [18] for calculation of the twist-4 function (2.6) with the following result

$$\phi_4^{asy}(u)=\frac{20}{3}\delta^2u\overline{u}\left[1-u(7-8u)\right]. \tag{3.13}$$

We use the function $\phi_4^{asy}(u)$ for calculation of the asymptotic twist-4 term.

IV. NUMERICAL RESULTS

In order to perform numerical computations we should fix various parameters appearing in the relevant expressions. Namely, we take the Borel parameter equal to $M^2=1\,\mathrm{GeV}^2$ and accept for the factorization and renormalization scales the values presented in Eq. (2.8). For the QCD coupling $\alpha_s(\mu_R^2)$ the two-loop expression with $\Lambda_3=0.34$ GeV is used. The value of the duality parameter $s_0=0.7\ \mathrm{GeV}^2$ is borrowed from the QCD sum rule for the correlator of two $\overline{u}\gamma_\mu\gamma_5 d$ currents [29]. The normalization scale is set equal to $\mu_0^2=1\ \mathrm{GeV}^2$.

Results of numerical computations are depicted in Figs. 3-6. As is seen (Fig. 3(a)) the leading twist LCSR contribution found employing $\varphi_{hol}(u)$, due to the broader shape of the HQCD distribution amplitude, is larger than the prediction obtained by means of $\varphi_{asy}(u)$. The twist-4 terms computed using the twist-4 function $\phi_4^{hol}(u)$ and the asymptotic version of Eq. (3.9) do not differ considerably from each other: one observes some deviation of the solid curve from the result of the standard QCD, i.e. from the dashed line. Here, the features of the different twist-4 terms that have interesting consequences should be emphasized. Thus, the asymptotic twist-4 term (the dot-dashed line) in the region of low momentum transfers demonstrates more rapid increasing on Q^2 than other twist-4 terms: in this region among the twist-4 contributions the HQCD prediction is a flattest and lowest one. Contrary, for the momentum transfers $Q^2>8\ \mathrm{GeV}^2$ the HQCD twist-4 term overtakes other twist-4 contributions. What is important, in the whole region of considering momentum transfers only the HQCD twist-4 term is monotonically increasing function of Q^2. All twist-4 terms are sensitive to the choice of the parameter δ_0^2. In Fig. 3(b), as an example, we plot the twist-2 and -4 terms generated by the HQCD distributions and by shaded area show sensitivity of the twist-4 term to δ_0^2.

The pion scaled electromagnetic FF $Q^2F_\pi(Q^2)$ computed employing the HQCD distribution amplitude and twist-4 function $\phi_4^{hol}(u)$ is depicted in Fig. 4(a). It is clear that the enhancement in the leading twist term is not sufficient to describe the existing data on $F_\pi(Q^2)$; the corresponding LCSR curve runs below the data points. If we replace the holographic twist-4 term by its asymptotic counterpart keeping the remaining ones unchanged, then the final expression describes the experimental results. Nevertheless such agreement can be achieved only for the large values of the parameter $\delta_0^2>0.2$. From this analysis it is legitimate to conclude that the enhancement generated by the shape of the function $\varphi_{hol}(u)$ in the relevant terms of Eq. (2.1) and the asymptotic twist-4 term is enough to explain the data on $F_\pi(Q^2)$ in the LCSR treatment.

The pion electromagnetic form factor was analyzed within the LCSR method in Ref. [24], where in calculations model DAs with two nonasymptotic terms and renormalon inspired twist-4 DAs were used, and from comparison with the data, constraints on the input parameters b_2^0 and b_4^0 were extracted. In the fitting proce-

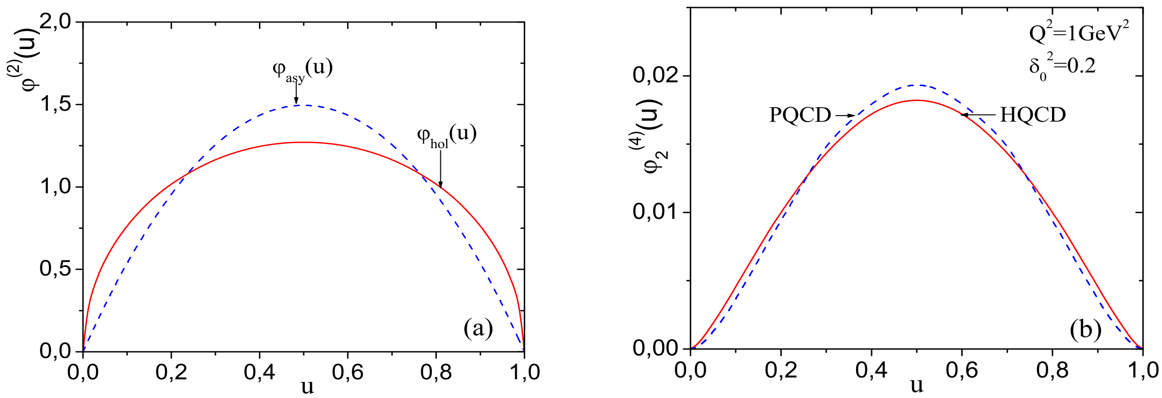

FIG. 1: **(a)** The leading twist distributions of the pion, **(b)** the twist-4 DAs obtained from $\varphi_{asy}(u)$ (the dashed curve) and $\varphi_{hol}(u)$ (the solid curve) by means of the renormalon approach.

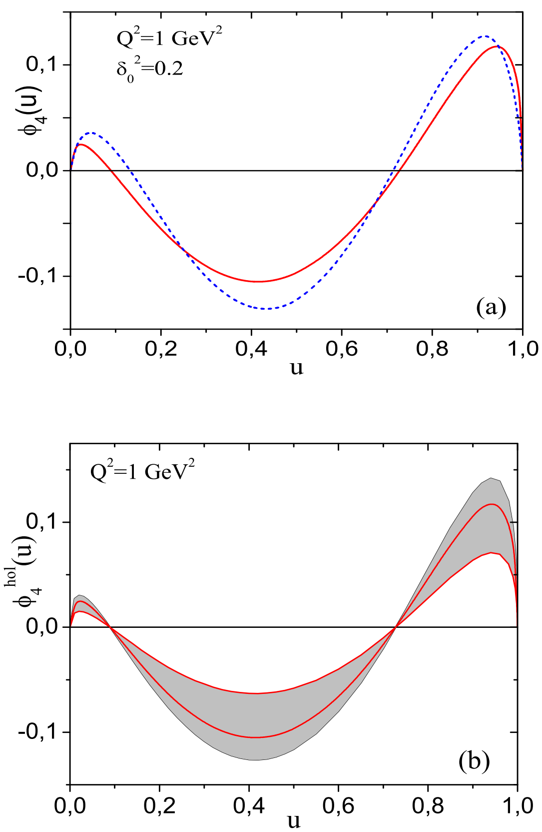

FIG. 2: **(a)** The twist-4 function $\phi_4(u)$ obtained in the renormalon-based model using the PQCD asymptotic DA (the dashed line) and the distribution amplitude of the HQCD (the solid line); **(b)** the dependence of the twist-4 function $\phi_4^{hol}(u)$ on the input parameter δ_0^2.

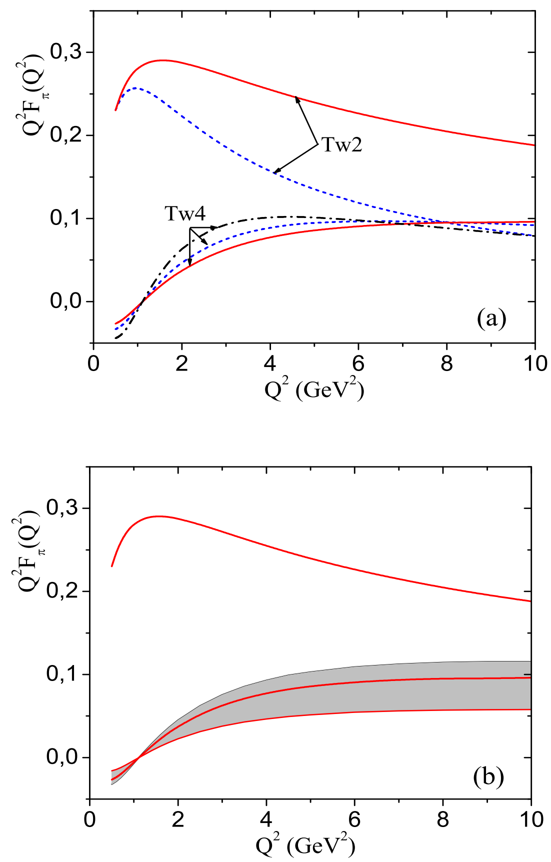

FIG. 3: **(a)** The leading and twist-4 contributions to the scaled pion form factor. In the calculations the PQCD DAs (the dashed lines) and DAs of the HQCD (the solid lines) are used. For comparison the asymptotic twist-4 term (the dot-dashed line) is also shown. All twist-4 terms are obtained using $\delta_0^2 = 0.2$. **(b)** The leading and twist-4 contributions to the FF found employing the functions $\varphi_{hol}(u)$ and $\phi_4^{hol}(u)$. The shaded area demonstrates the allowed values of the twist-4 term obtained by varying the parameter δ_0^2 within the limits (3.7).

dure in Ref. [24] both the old experimental results [30] and new ones from the Jefferson Lab. F_π Collaboration [31] were used. It is worth noting that the pion form factor is not measured directly and real measurements of

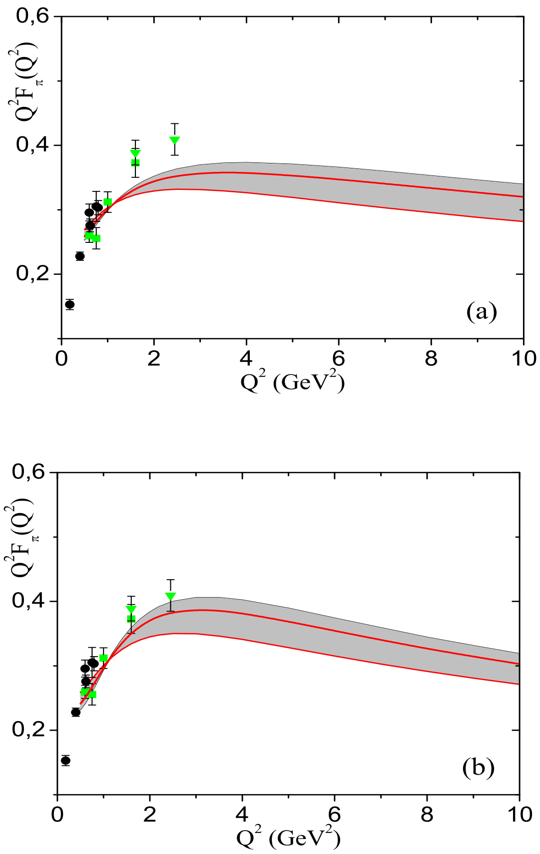

FIG. 4: **(a)** The pion scaled FF as a function of Q^2. It is computed using the HQCD distribution amplitude $\varphi_{hol}(u)$ and the renormalon-based twist-4 function $\phi_4^{hol}(u)$. The shaded area shows allowed limits of the form factor. For the central solid line $\delta_0^2 = 0.2$. **(b)** The pion FF obtained using the asymptotic twist-4 term and the holographic ones for the remaining contributions from Eq. (2.1). For the central solid line $\delta_0^2 = 0.2$. Concerning the data - see discussion in the text .

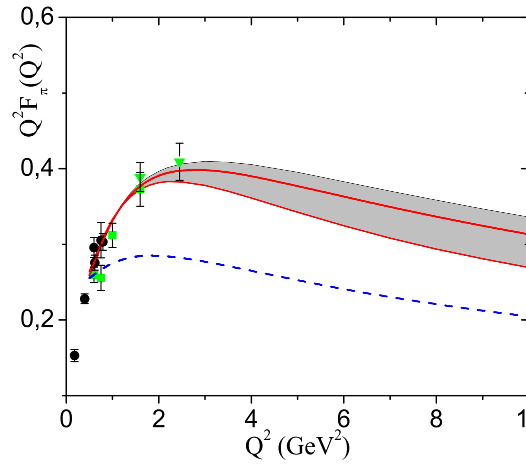

FIG. 5: The pion scaled FF as a function of Q^2. The central solid line is the LCSR prediction obtained employing the PQCD leading twist DA with $b_2^0 = 0.25$, $b_4^0 = 0$ and corresponding renormalon-based twist-4 function $\phi_4(u, Q^2)$. The shaded area is obtained by varying δ_0^2 within the allowed limits at fixed $b_2^0 = 0.25$, $b_4^0 = 0$; for the central line $\delta_0^2 = 0.2$. For comparison the LCSR result found using $\varphi_{asy}(u)$ is also shown (the dashed line, $\delta_0^2 = 0.2$).

the process $\gamma^* p \to \pi^+ n$ require an extrapolation of the off-shell pion to the mass-shell. Because this extrapolation becomes increasingly problematic as Q^2 increases and at high momentum transfers the old results suffer from large uncertainties, in the present work we remove them from consideration. We also correct the data from the F_π Collaboration reported in Ref. [32], and add two new points extracted by $F_{\pi 2}$ Collaboration [33]: in Figs. 4, 5 and 6 the rectangle and triangle symbols denote the F_π and $F_{\pi 2}$ data, respectively. Strictly speaking, the LCSR method is applicable in the domain of momentum transfers $Q^2 \geq 1$ GeV^2. Nevertheless, we extend our numerical calculations to the region $Q^2 < 1$ GeV^2 to reveal properties of the leading and twist-4 terms at low momentum transfers. The data below this border (the solid circles) are included into the figures for illustrative purposes and are irrelevant for our present discussions.

We have repeated calculations of Ref. [24] using the pion model DAs with two nonasymptotic terms. One of these FFs, that corresponds to values of the input parameters $b_2^0 = 0.25$, $b_4^0 = 0$ and $\delta_0^2 = 0.2$, is plotted in Fig. 5. The achieved agreement with new data of the JLAB experiment is encouraging. It is easy to see that excluding of the unprecise data from analysis and adding new ones results in a shift of the parameter b_2^0 towards larger values than in Ref. [24]. The similar curves can be obtained in the more general case $b_2^0 \neq 0$, $b_4^0 \neq 0$ as well.

In the framework of the holographic QCD the pion electromagnetic FF has been calculated in Refs. [11, 12, 13]. The soft-wall model expression for the form factor is especially simple, and is given by the monopole form [12]

$$F_\pi(Q^2) = \frac{4k^2}{4k^2 + Q^2}, \tag{4.1}$$

where $k = 0.375\,\mathrm{GeV}$.

In Fig. 6 we compare the LCSR predictions found using the functions $\varphi_{hol}(u)$ and $\phi_4^{hol}(u)$ (the lower solid line) and $\varphi_{hol}(u)$ and $\phi_4^{asy}(u)$ (the dashed line) with the prediction of the soft-wall HQCD (the upper solid line). Let us note that the dashed line has been obtained choosing the maximal allowed value of $\delta_0^2 = 0.24$.

The first LCSR result with the HQCD distribution amplitude and renormalon inspired twist-4 function in the range of the momentum transfers $2 \leq Q^2 \leq 10$ GeV^2 demonstrates the scaling in accordance with $Q^2 F_\pi(Q^2) \simeq$ const law with a weak sign of the logarithmic running of the QCD coupling. This feature can be explained as a consequence of the dependence of the corresponding twist-4 term on Q^2 due to the enhancement of the

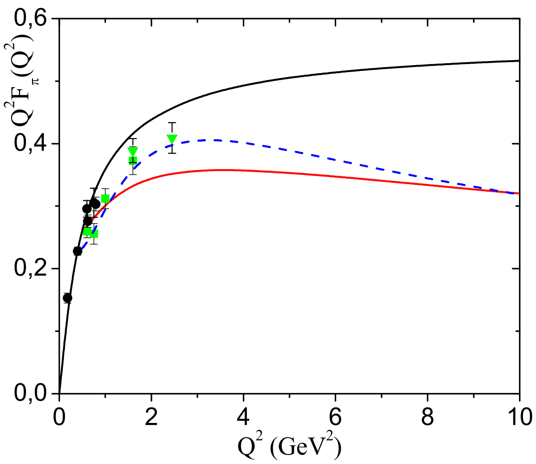

FIG. 6: The pion scaled FF as a function of Q^2. The lower solid line is prediction of the LCSR method with the distribution amplitude of the HQCD. The dashed line is the same the HQCD result, but the HQCD twist-4 term replaced by the asymptotic twist-4 one. The upper solid line is the soft-wall holographic QCD prediction from Ref. [12].

$\varphi_4^{hol}(u)$ at end-point regions. The second LCSR prediction computed also in this work differs from the first one only in the choice of the twist-4 term. It is evident that the dashed line shows more rapid fall-off with Q^2 than the lower solid line, though due to the choice $\delta_0^2 = 0.24$ is in agreement with the data. Such rapid falling is typical for other LCSR curves obtained using the PQCD DAs (Fig. 5).

In the LCSR method the soft component of the pion FF is modeled by the leading order twist-2 term and some piece of the $O(\alpha_{\rm s})$ order correction to the twist-2 term. The leading order higher twist terms are soft contributions to the pion FF as well. There are various approaches to model the soft component of the FF. For example, in Ref. [34] the pion FF was evaluated applying the standard hard-scattering approach and the QCD running coupling method. It was demonstrated that the Borel resummed expression of the form factor contains both the soft and hard components, and in the asymptotic limit $Q^2 \to \infty$ the standard PQCD result can be recovered. Alternatively, in Ref. [35] soft and hard contributions to the pion electromagnetic FF were calculated within the light-front quark model and comparison with the AdS/CFT prediction was performed. The authors reported on excellent agreement of their result with the HQCD prediction.

Comparison of our results with the soft-wall holographic QCD prediction reveals the discrepancy between them. First of all, the prediction of the holographic QCD overestimates the JLAB data: a situation with a hard-wall result is even worse [12]. Second, the HQCD prediction obeys the scaling $Q^nF_\pi(Q^2) \to$ const with $n \neq 2$. From our point of view, emphasized also in Ref. [35], for a reliable analysis one should evolve the pion DA to a scale of the considering process, which in our case is equal to a few GeV^2. This procedure inevitably demands to include nonasymptotic terms into the holographic DA of the pion. In other words, for credible phenomenological applications the holographic QCD version of Eq. (3.2), where the function $\varphi_{hol}(u)$ is only the first term in a relevant expansion, is required. Higher order corrections neglected in present investigations of the AdS/CFT correspondence may improve a situation, too [35].

For successful applications of the pion HQCD distribution amplitude within the framework of traditional methods of the perturbative QCD, one should prove that this DA describes also other exclusive and semi-inclusive processes involving the pion. The simplest such process is the pion electromagnetic transition form factor $F_{\pi\gamma}(Q^2)$. In the context of the LCSR method it was computed in Refs. [36, 37]: the result was derived with twist-4 accuracy including the next-to-leading order correction to the twist-2 term. In the context of the LCSR and renormalon methods this form factor was analyzed in Refs. [28, 38]. Preliminary calculations demonstrate that the twist-2 term in the LCSR expression for $F_{\pi\gamma}(Q^2)$ found using $\varphi_{hol}(u)$ exceeds the measured experimental data [39]. But contributions of the next-to-leading order and twist-4 terms to the LCSR are negative and reduce the magnitude of the leading twist contribution (see, for example, [28]): hence an agreement with the data may be achieved. A complete analysis of $F_{\pi\gamma}(Q^2)$ requires to compute using $\varphi_{hol}(u)$ the renormalon-based twist-4 function $\Phi^{(4)}(u, Q^2)$ [28, 36] that determines the twist-4 contribution to the transition FF. This problem requires separate investigations.

V. CONCLUSIONS

In the present work we have calculated the pion spacelike electromagnetic form factor $F_\pi(Q^2)$ in the QCD LCSR framework. We have used the distribution amplitude of the pion derived in the holographic QCD, which has a broader shape than the PQCD asymptotic DA. The twist-4 DA has been obtained applying the renormalon inspired model of Ref. [23]. The prediction of the LCSR method obtained with the twist-6 accuracy by means of the functions $\varphi_{hol}(u)$ and $\phi_4^{hol}(u)$ lies below the data, and does not describe the experimental situation. The agreement with the data can be achieved provided the HQCD twist-4 term is replaced by the asymptotic one and $\delta_0^2 > 0.2$ is chosen.

We have also computed $F_\pi(Q^2)$ utilizing the pion PQCD leading twist DAs with two nonasymptotic terms and corresponding renormalon inspired twist-4 DAs. It has been proved that in this case in the region $Q^2 >$ 1 GeV^2 the QCD LCSR method explains the new experimental situation emerged due to the data of the JLAB experiment.

Comparison with results of the holographic QCD [11, 12, 13] has revealed an interesting tendency: the holo-

graphic QCD predictions for $Q^2F_\pi(Q^2)$ obtained within the both hard- and soft-wall models are increasing functions of Q^2, whereas in the LCSR treatment at large momentum transfers one finds an opposite picture. It is worth noting that the scaling of the form factor calculated employing the functions $\varphi_{hol}(u)$ and $\varphi_4^{hol}(u)$ is very promising. Unfortunately, the corresponding curve runs below the data. In order to enhance the magnitude of $F_\pi(Q^2)$ and reach an agreement with the data additional contributions are needed. It seems to us that such contributions may appear due to the holographic QCD counterparts of the nonasymptotic terms in the pion PQCD leading twist DA (3.2). If extracted, these terms may improve scaling properties and normalization of the holographic QCD predictions themselves.

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